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Effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system

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Abstract The objective of this paper is to study the effects of Hall current and radiation absorption on MHD free convection mass transfer flow of a micropolar fluid in a rotating frame of reference. A uniform magnetic field acts perpendicular to the porous surface in which absorbs micropolar fluid with a constant suction velocity. The entire system rotates about the axes normal to the plate with uniform angular velocity Ω. The dimensionless governing equations for this investigation are reduced to a system of linear differential equations using regular perturbation method, and equations are solved analytically. The influence of various flow parameters of the flow field has been discussed and explained graphically. The present study is of immediate interest in geophysical, cosmically fluid dynamics, medicine, biology, and all those processes which are greatly embellished by a strong magnetic field with a low density of the gas.

1. Introduction

Micropolar fluids are fluids with microstructure and asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. These types of fluids are used in analyzing liquid crystals, animal blood, fluid flowing in brain, exotic lubricants, the flow of colloidal suspensions, paints, turbulent shear flows, and body fluids after mathematically and industrially [1,2]. The theory of micropolar fluids was developed by Eringen [3,4]. The comprehensive literature on micropolar fluids, thermomicropolar fluids and their applications in engineering and technology were presented by [5,6]. Srinivasacharya et al. [7] analyzed the unsteady stokes flow of micropolar fluid between two parallel porous plates. Muthuraj and Srinivas [8] investigated fully developed MHD flow of a micropolar and viscous fluid in a vertical porous space using HAM.

The study of heat and mass transfer for an electrically conducting micropolar fluid past a porous plate under the influence of a magnetic field in a rotating frame of reference has attracted the interest of many investigators in view of its applications in many industrial (in the design of turbines and turbo mechanics), astrophysical (dealing with the sunspot development, the solar cycle and the structure of a rotating magnetic stars), geophysical (hydrologists to study the migration of the underground water, petroleum engineers to observe the

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Nomenclature

B_0	applied magnetic field
C	concentration of the solute
C_f	skin friction coefficient
C_p	specific heat at constant pressure
C_s	couple stress coefficient
C_w	concentration of the solute at the plate
C_∞	concentration of the solute far away from the plate
D_m	molecular diffusivity
en_e	electron charger
F	heat radiation parameter
g	acceleration due to gravity
G_m	modified Grashof number
G_r	Grashof number
$\overline{H}$	magnetic field strength
H_0	the externally applied transverse magnetic field
i	imaginary unit
j	micro inertia density
$\overline{J}$	current strength
K	permeability of porous medium
k	thermal conductivity of the fluid
k^*	mean absorption coefficient
M	magnetic field parameter
m	hall parameter
n	frequency of oscillation
n_e	number electron density
Nu	Nusselt number
P_e	electron pressure
P_r	Prandtl number
Q	additional heat source
Q_l	radiation absorption parameter
q_r	radiative heat flux
R	rotational parameter
Re_x	local Reynolds number
R_r	chemical reaction rate constant
S	suction parameter
S_c	Schmidt number
Sh_x	Sherwood number
T	temperature of the fluid in the boundary layer
t	non-dimensional time
T_w	wall temperature of the fluid
T_∞	temperature of the fluid far away from the plate
U_r	uniform reference velocity
$\overline{V}$	velocity vector
w_0	normal velocity
(u, v, w)	velocity components
(x, y, z)	Cartesian co-ordinates
$(\bar{\omega}_1, \bar{\omega}_2)$	micro-rotation components

Greek symbols

α	chemical reaction parameter
β_C	coefficient of concentration expansion
β_T	coefficient of thermal expansion
Δ	viscosity ratio
υ	kinematic viscosity
υ_r	kinematic micro-rotation viscosity
Λ	spin gradient velocity
Ω	angular velocity
λ	dimensionless material parameter
μ	coefficient of viscosity
μ_e	magnetic permeability
σ	electrical conductivity of the fluid
σ^*	Stefan-Boltzmann constant parameter
ε	small constant quantity
ρ	density of the fluid
τ_e	electron collision time
ω	micro-rotation profile
ω_e	electron frequency
ϕ	non-dimensional concentration
θ	non-dimensional temperature

Subscripts

w	condition at the wall
∞	condition at free stream

movement of oil and gas through the reservoir), and many other practical applications, that is, in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet). It is well known that a number of astronomical bodies possess fluid interiors and magnetic fields. Changes that take place in the rate of rotation suggest the possible importance of hydromagnetic spin-up. Also rotating heat exchangers are extensively used by the chemical and automobile industries. The pioneering works [9,10] have laid the foundation stone in this field. Many authors have studied the flow and heat transfer in a rotating system with various geometrical situations [11–15].

The combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling the tower, and the flow in a desert cooler, the heat and mass transfer occurs simultaneously. Damesh et al. [16] have studied the combined effect of heat generation or absorption and first order chemical reaction to micropolar fluid flows over a uniform stretched surface. Some other related works can also be found in the papers [17–19].

In all the previous investigations, the effect of thermal radiation on the flow and heat transfer has not been provided. The effect of radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperatures, radiation effect can be quite significant, many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer become very important for design of reliable equipment, nuclear plants, gas turbines and various propulsion devices or aircrafts, missiles, satellites, and space vehicles. Abo-eldohad and Ghonaim [20] analyzed the radiation effects on heat transfer of a micropolar fluid through a porous medium. Rahman and Sultana [21] have studied the steady convective flow of a micropolar fluid past a vertical porous flat plate in the presence of radiation with variable heat flux in porous medium. The effects of thermal radiation were also investigated by the researchers [22,23].

From the scientific point of observation, flow arising from temperature and material difference is applied in chemical engineering, geothermal reservoirs, aeronautics and astrophysics. In some applications, magnetic forces are present, and at other times, the flow is further complicated by the presence of radiation absorption, an excellent paradigm of this is in the planetary atmosphere where there is radiation absorption from nearby stars. The influence of a magnetic field on the flow of an electrically conducting viscous fluid with mass transfer and radiation absorption is also useful planetary atmosphere research [24]. Umavathi and Malashetty [25] have studied the problem of combined free and forced mixed convection flow in a vertical channel with symmetric and asymmetric boundary heating in the presence of viscous and Joulean dissipations. The effects of chemical reaction and radiation absorption in the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with a heat source and suction were studied by [26,27].

The current development of magnetohydrodynamics application is toward a strong magnetic field (so that the influence of electromagnetic force is noticeable) and toward a low density of the gas (such as in space flight and in nuclear fusion research). Under this condition, the Hall current becomes important. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in geophysical, cosmical fluid dynamics, medicine and biology. Application in biomedical engineering includes cardiac MRI, ECG, etc. Several engineering applications in areas of Hall accelerator as well as in flight MHD have been studied by the researchers [28–31]. Recently, Bakr [32] presented an analysis of MHD free convection and mass transfer adjacent to moving vertical plate for micropolar fluid in a rotating frame of reference in the presence of heat generation/absorption and a chemical reaction. Later, the same work was extended by Das [33] with thermal radiation effects.

To the best of our knowledge, the problem of magnetohydrodynamics unsteady free convection flow of a micropolar fluid in a rotating frame of reference to Hall effects and radiation absorption has remained unexplored. So, the main objective of this paper is to extend the work of Ref. [33] in three directions: (i) to consider MHD micropolar fluid in a rotating frame of reference, (ii) to consider the Hall effects, and (iii) to include the radiation absorption. The governing equations of the flow are solved analytically, and the effects of various flow parameters on the flow field have been discussed. The organization of the remnants of the paper is as follows. In Section 2, we describe the model with it governing equations and boundary conditions. Here, we also describe the solution method briefly. In Section 3, we present results and discussion. Finally, in Section 4, we summarize our results and present our conclusions. A comparison is made with the available results in the literature, and salient features of the new results are analyzed and discussed.

2. Mathematical analysis

In the Cartesian coordinate system (x, y, z), we consider an unsteady three dimensional flow of an incompressible viscous electrically conducting micropolar fluid past a semi-infinite vertical porous plate in a rotating system with thermal radiation, chemical reaction, and Hall current in the presence of a uniform transverse magnetic field. The flow is assumed to be in the x-direction, which is taken along the plate in upward direction, and the y-axis is normal to it and the z-axis along the width of the plate as shown in Fig. 1. The fluid is considered to be a gray, emitting, and absorbing heat, but non-scattering medium. When the strength of the magnetic field is very large, the generalized Ohm's law in the absence of electric field takes the following form.

$$\overline{J} + \frac{\omega_e \tau_e}{B_0} \overline{J} \times \overline{H} = \sigma\left(\mu_e \overline{V} \times \overline{H} + \frac{1}{e n_e} \nabla P_e\right) \tag{1}$$

Under the assumption that the electron pressure (for weakly ionized gas), the thermo-electric pressure and ion-slip conditions are negligible, now the above equation becomes:

$$j_x = \frac{\sigma \mu_e H_0}{1+m^2}(mv - u) \quad \text{and } j_z = \frac{\sigma \mu_e H_0}{1+m^2}(mu + v) \tag{2}$$

where u is the x-component of $\overline{V}$, v is the y-component of $\overline{V}$ and $m(=\omega_e \tau_e)$ is Hall parameter.

Our investigation is restricted to the following assumptions.

(i) All the fluid properties except the density in the buoyancy force terms are constant.
(ii) The plate is electrically non-conducting.
(iii) The entire system is rotating with angular velocity Ω about the normal to the plate.
(iv) The magnetic Reynolds number is so small that the induced magnetic field can be neglected. Also, the electrical conductivity σ of the fluid is reasonably low, and hence, the Ohmic dissipation may be neglected.
(v) It is assumed that there is no applied voltage which implies that electric field is absent.

The equations governing the flow, heat, and mass transfer are:

$$\frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega v = (\upsilon + \upsilon_r)\frac{\partial^2 u}{\partial z^2} + g\beta_T(T - T_\infty) \\ + g\beta_C(C - C_\infty) - \frac{\upsilon u}{k} - \upsilon_r \frac{\partial \bar{\omega}_2}{\partial z} \\ + \frac{\sigma \mu_e^2 H_0^2 (mv - u)}{\rho(1+m^2)}\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + w\frac{\partial v}{\partial z} + 2\Omega u = (\upsilon + \upsilon_r)\frac{\partial^2 v}{\partial z^2} - \frac{\upsilon v}{k} + \upsilon_r \frac{\partial \bar{\omega}_1}{\partial z} \\ - \frac{\sigma \mu_e^2 H_0^2 (mu + v)}{\rho(1+m^2)}\end{aligned} \tag{5}$$

$$\frac{\partial \bar{\omega}_1}{\partial t} + w\frac{\partial \bar{\omega}_1}{\partial z} = \frac{\Lambda}{\rho j}\frac{\partial^2 \bar{\omega}_1}{\partial z^2} \tag{6}$$

$$\frac{\partial \bar{\omega}_2}{\partial t} + w\frac{\partial \bar{\omega}_2}{\partial z} = \frac{\Lambda}{\rho j}\frac{\partial^2 \bar{\omega}_2}{\partial z^2} \tag{7}$$

$$\begin{aligned}\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial z^2} - \frac{Q}{\rho C_P}(T - T_\infty) + \frac{Q_l^*}{\rho C_P}(C - C_\infty) \\ - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial z}\end{aligned} \tag{8}$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D_m \frac{\partial^2 C}{\partial z^2} - R_r(C - C_\infty) \tag{9}$$

The relevant boundary conditions are:

$$u = v = 0, \bar{\omega}_1 = \bar{\omega}_2 = 0, T = T_\infty, C = C_\infty \quad \text{For } t \leqslant 0 \tag{10}$$

$$u = U_r\left\{1 + \frac{\varepsilon}{2}(e^{\text{int}} + e^{-\text{int}})\right\}, \quad v = 0$$

$$\bar{\omega}_1 = -\frac{1}{2}\frac{\partial v}{\partial z}, \bar{\omega}_2 = \frac{1}{2}\frac{\partial u}{\partial z}, T = T_w, C = C_w \quad \text{at } z = 0 \quad \text{For } t > 0 \tag{11}$$

$$u = v = 0, \bar{\omega}_1 = \bar{\omega}_2 = 0, T = T_\infty, C = C_\infty \quad \text{as } z \to \infty$$

The oscillatory plate velocity assumed in Eq. (11) is based on the suggestion proposed by Ganapathy [34].

We now consider a convenient solution of the continuity Eq. (3) to be

$$w = -w_0 \tag{12}$$

where the w_0 represents the normal velocity at the plate which is positive for suction and negative for blowing.

The radiative heat flux term by using the Rosseland approximation is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z} \tag{13}$$

$$T^4 = 4TT_\infty^3 - 3T_\infty^4 \tag{14}$$

$$\frac{\partial q_r}{\partial z} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial z^2} \tag{15}$$

Let us introduce the following non-dimensional quantities:

$$\begin{gathered} u^* = \frac{u}{U}, v^* = \frac{v}{U_r}, z^* = \frac{zU_r}{\upsilon}, t^* = \frac{tU_r^2}{\upsilon}, n^* = \frac{n\upsilon}{U_r^2}, Q_l = \frac{Q_l^*(C_w - C_\infty)}{(T_w - T_\infty)U_r^2} \\ \bar{\omega}_1^* = \frac{\bar{\omega}_1\upsilon}{U_r^2}, \bar{\omega}_2^* = \frac{\bar{\omega}_2\upsilon}{U_r^2}, \alpha = \frac{R_r\upsilon}{U_r^2}, M = \frac{\mu_e H_0}{U_r}\sqrt{\frac{\sigma\upsilon}{\rho}}, \lambda = \frac{\Lambda}{\mu j}, \Delta = \frac{\upsilon_r}{\upsilon} \\ R = \frac{2\Omega\upsilon}{U_r^2}, K = \frac{kU_r^2}{\upsilon^2}, F = \frac{4T_\infty^3\sigma}{kk^*}, S = \frac{w_0}{U_r}, Sc = \frac{\upsilon}{D_m}, Pr = \frac{\mu C_p}{k} \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty}, Gr = \frac{\upsilon g\beta_T(T_w - T_\infty)}{U_r^3}, \\ Gm = \frac{\upsilon g\beta_c(C_w - C_\infty)}{U_r^3}, Q^* = \frac{Q\upsilon^2}{U_r^2 k} \end{gathered} \tag{16}$$

In view of Eq. (16), the basic field Eqs. (3)–(9) can be expressed in non-dimensional form as:

$$\begin{aligned} \frac{\partial u}{\partial t} - S\frac{\partial u}{\partial z} - Rv = (1+\Delta)\frac{\partial^2 u}{\partial z^2} + Gr\theta + Gm\phi \\ - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)u - \Delta\frac{\partial \bar{\omega}_2}{\partial z} \\ + \left(\frac{mM^2}{1+m^2}\right)v \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial v}{\partial t} - S\frac{\partial v}{\partial z} + Ru = (1+\Delta)\frac{\partial^2 v}{\partial z^2} - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)v \\ + \Delta\frac{\partial \bar{\omega}_1}{\partial z} - \left(\frac{mM^2}{1+m^2}\right)u \end{aligned} \tag{18}$$

$$\frac{\partial \bar{\omega}_1}{\partial t} - S\frac{\partial \bar{\omega}_1}{\partial z} = \lambda\frac{\partial^2 \bar{\omega}_1}{\partial z^2} \tag{19}$$

$$\frac{\partial \bar{\omega}_2}{\partial t} - S\frac{\partial \bar{\omega}_2}{\partial z} = \lambda\frac{\partial^2 \bar{\omega}_2}{\partial z^2} \tag{20}$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial z} = \frac{1}{Pr}(1 + \frac{4F}{3})\frac{\partial^2 \theta}{\partial z^2} - \frac{Q}{Pr}\theta + Q_l\phi \tag{21}$$

$$\frac{\partial \phi}{\partial t} - S\frac{\partial \phi}{\partial z} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial z^2} - \alpha\phi \tag{22}$$

The corresponding boundary conditions (10) and (11) in view of scaling relation (16) reduce to

$$u = v = 0, \bar{\omega}_1 = \bar{\omega}_2 = 0, \theta_0 = 0, \phi_0 = 0 \qquad \text{For } t \leqslant 0 \tag{23}$$

$$\begin{gathered} u = 1 + \frac{\varepsilon}{2}(e^{\text{int}} + e^{-\text{int}}), v = 0, \bar{\omega}_1 = -\frac{1}{2}\frac{\partial v}{\partial z}, \bar{\omega}_2 = \frac{1}{2}\frac{\partial u}{\partial z}, \\ \theta = 1, \phi = 1 \quad \text{at } z = 0 \qquad \text{For } t > 0 \\ u = v = 0, \bar{\omega}_1 = \bar{\omega}_2 = 0, \theta = 0, \phi = 0 \quad \text{as } z \to \infty \end{gathered} \tag{24}$$

To obtain desired solutions, we now simplify Eqs. (17)–(20) by putting the fluid velocity and angular velocity in the complex form as:

$V = u + iv, \omega = \bar{\omega}_1 + i\bar{\omega}_2$ and we get

$$\begin{aligned} \frac{\partial V}{\partial t} - S\frac{\partial V}{\partial z} + iRV = (1+\Delta)\frac{\partial^2 V}{\partial z^2} + Gr\theta + Gm\phi \\ - \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right)V + i\Delta\frac{\partial \omega}{\partial z} \\ - i\left(\frac{mM^2}{1+m^2}\right)V \end{aligned} \tag{25}$$

$$\frac{\partial \omega}{\partial t} - S\frac{\partial \omega}{\partial z} = \lambda\frac{\partial^2 \omega}{\partial z^2} \tag{26}$$

The associated boundary conditions (23) and (24) become

$$V = 0, \omega = 0, \theta = 0, \phi = 0 \quad \text{For } t \leqslant 0 \tag{27}$$

$$V = 1 + \frac{\varepsilon}{2}(e^{\text{int}} + e^{-\text{int}}), \omega = \frac{i}{2}\frac{\partial V}{\partial z}, \theta = 1, \phi = 1 \text{ at } z = 0 \text{ and } V = 0, \omega = 0, \theta = 0, \phi = 0 \text{ as } z \to \infty \quad \text{For } t > 0 \tag{28}$$

Analytical solutions:

In order to reduce the above system of partial differential equations in dimensionless form, we may represent the linear and angular velocities, temperature and concentration as V, ω, θ, and ϕ as

$$\begin{aligned} V(z,t) &= V_0 + \frac{\varepsilon}{2}\{e^{\text{int}}V_1(z) + e^{-\text{int}}V_2(z)\} \\ \omega(z,t) &= \omega_0 + \frac{\varepsilon}{2}\{e^{\text{int}}\omega_1(z) + e^{-\text{int}}\omega_2(z)\} \\ \theta(z,t) &= \theta_0 + \frac{\varepsilon}{2}\{e^{\text{int}}\theta_1(z) + e^{-\text{int}}\theta_2(z)\} \\ \phi(z,t) &= \phi_0 + \frac{\varepsilon}{2}\{e^{\text{int}}\phi_1(z) + e^{-\text{int}}\phi_2(z)\} \end{aligned} \tag{29}$$

Substituting the above Eq. (29) into the Eqs. (19), (20), (23), and (24) and equating the harmonic and non-harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$, we obtain the following set of equations:

Zeroth order equations are:

$$(1+\Delta)V_0'' + SV_0' - a_1 V_0 + Gr\theta_0 + Gm\phi_0 + i\Delta\omega_0' = 0 \quad (30)$$
$$\lambda\omega_0'' + S\omega_0' = 0 \quad (31)$$
$$(3+4F)\theta_0'' + 3SPr\theta_0' - 3Q\theta_0 + 3Q_l Pr\phi_0 = 0 \quad (32)$$
$$\phi_0'' + SSc\phi_0' - \alpha Sc\phi_0 = 0 \quad (33)$$

First order equations are:

$$(1+\Delta)V_1'' + SV_1' - a_2 V_1 + Gr\theta_1 + Gm\phi_1 + i\Delta\omega_1' = 0 \quad (34)$$
$$\lambda\omega_1'' + S\omega_1' - in\omega_1 = 0 \quad (35)$$
$$(3+4F)\theta_1'' + 3SPr\theta_1' - 3(Q + inPr)\theta_1 + 3Q_l Pr\phi_1 = 0 \quad (36)$$
$$\phi_1'' + SSc\phi_1' - (\alpha + in)\phi_1 = 0 \quad (37)$$

Second order equations are:

$$(1+\Delta)V_2'' + SV_2' - a_3 V_2 + Gr\theta_2 + Gm\phi_2 + i\Delta\omega_2' = 0 \quad (38)$$
$$\lambda\omega_2'' + S\omega_2' + in\omega_2 = 0 \quad (39)$$
$$(3+4F)\theta_2'' + 3SPr\theta_2' - 3(Q - inPr)\theta_2 + 3Q_l Pr\phi_2 = 0 \quad (40)$$
$$\phi_2'' + SSc\phi_2' - (\alpha - in)\phi_2 = 0 \quad (41)$$

where the prime denote differentiation with respect to z.

The corresponding boundary conditions can be written as

$$\begin{aligned} &V_0 = V_1 = V_2 = 1, \omega_0 = \frac{i}{2}V_0', \omega_1 = \frac{i}{2}V_1', \omega_2 = \frac{i}{2}V_2' \\ &\theta_0 = 1, \theta_1 = \theta_2 = 0, \phi_0 = 1, \phi_1 = \phi_2 = 0, \quad at\ z = 0 \end{aligned} \quad (42)$$

$$\begin{aligned} &V_0 = V_1 = V_2 = 0, \omega_0 = \omega_1 = \omega_2 = 0 \\ &\theta_0 = \theta_1 = \theta_2 = 0, \phi_0 = \phi_1 = \phi_2 = 0 \quad at\ z \to \infty \end{aligned} \quad (43)$$

Solving Eqs. (30)–(41) under the boundary conditions (42) and (43) we obtain the expression for translational velocity, micro-rotation, temperature and concentration as:

$$\begin{aligned} V = {} & A_3 e^{-m_1 z} + A_4 e^{-m_2 z} + A_5 e^{-m_3 z} + A_6 e^{-\frac{S}{\lambda}z} + \frac{\varepsilon}{2}\{(A_7 e^{-m_4 z} \\ & + A_8 e^{-m_5 z})e^{\text{int}} + (A_9 e^{-m_6 z} + A_{10} e^{-m_7 z})e^{-\text{int}}\} \end{aligned} \quad (44)$$

$$\omega = B_1 e^{-\frac{S}{\lambda}z} + \frac{\varepsilon}{2}\{B_2 e^{\text{int} - m_4 z} + B_3 e^{-(\text{int} + m_6 z)}\} \quad (45)$$

$$\theta = A_1 e^{-m_1 z} + A_2 e^{-m_2 z} \quad (46)$$

$$\phi = e^{-m_1 z} \quad (47)$$

The physical quantities of engineering interest are skin friction coefficient, couple stress coefficient, Nusselt number and Sherwood number.

Skin friction is caused by viscous drag in the boundary layer around the plate, and then, it is important to discuss the skin friction, from the knowledge of velocity, free skin frictionsional form can be calculated as follows:

$$\begin{aligned} C_f &= \frac{\tau_{w|_{z=0}}}{\rho U_r^2} = \left\{1 + \Delta\left(1 + \frac{i}{2}\right)\right\}V'(0) \\ &= -\left\{1 + \Delta\left(1 + \frac{i}{2}\right)\right\}[A_3 m_1 + A_4 m_2 + A_5 m_3 + \frac{S}{\lambda}A_6 \\ &\quad + \frac{\varepsilon}{2}\{(A_7 m_4 + A_8 m_5)e^{\text{int}} + (A_9 m_6 + A_{10} m_7)e^{-\text{int}}\}] \end{aligned} \quad (48)$$

The couple stress coefficient at the wall C_s is given by

$$\begin{aligned} C_s &= \left.\frac{\partial\omega_1}{\partial z}\right|_{z=0} + i\left.\frac{\partial\omega_2}{\partial z}\right|_{z=0} = \omega'(0) \\ &= -\left\{\frac{SB_1}{\lambda} + \frac{\varepsilon}{2}(B_2 m_4 e^{\text{int}} + B_3 m_6 e^{-\text{int}})\right\} \end{aligned} \quad (49)$$

The rate of heat transfer between the fluid and the plate is studied through non-dimensional Nusselt number. The rate of heat transfer in terms of Nusselt number is given by

$$\begin{aligned} N_u &= \frac{\chi\left(\frac{\partial T}{\partial z}\right)_{z=0}}{T_w - T_\infty} = -Re_x\theta'(0) \Rightarrow \frac{N_u}{Re_x} = -\theta'(0) \\ &= A_1 m_1 + A_2 m_2 \end{aligned} \quad (50)$$

where $Re_x = \frac{U_r x}{\upsilon}$ is the local Reynolds number.

The local Sherwood number Sh_x is given by

$$Sh_x = -\frac{\chi\left(\frac{\partial C}{\partial z}\right)_{z=0}}{C_w - C_\infty} = -Re_x\phi'(0) \Rightarrow \frac{Sh_x}{Re_x} = -\phi'(0) = m_1 \quad (51)$$

3. Results and discussion

Numerical evaluation of analytical results reported in the previous section was performed, and a representative set of results is reported graphically in Figs. 1-11. These results are obtained to illustrate the influence of various parameters on the velocity V, micro-rotation ω, temperature θ, and concentration ϕ profiles, etc. In these calculations, we consider $\varepsilon = 0.02$, $n = 10$, and $t = 0.1$, while other parameters are varied over a range which are listed in figure captions.

To verify the validity and accuracy of the present analysis, our results have been compared to the local skin friction coefficient, the local Nusselt number, and Sherwood number with those obtained by [33] for various values of Δ. The results of this comparison are given in Table 1. It can be seen from this table that excellent agreement between the results exists. This favorable comparison leads confidence in the next section.

Figs. 2a and 2b, respectively, show the translational velocity and micro-rotation profiles for different values of a magnetic field parameter M. Fig. 2a shows that the translational velocity field decreases with increase in a magnetic field parameter M along the surface. These effects are much stronger near the surface of the plate. This indicates that the fluid velocity reduced by increasing the magnetic field and confines the fact

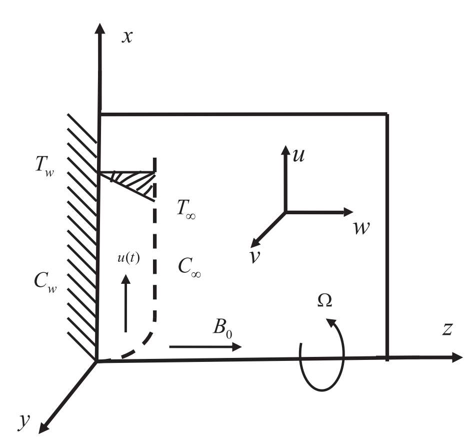

Figure 1 Physical model and coordinate system of the problem.

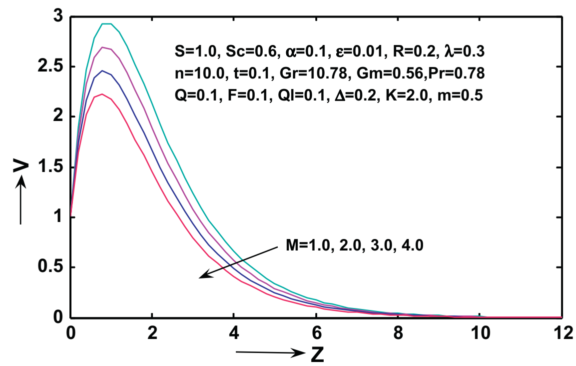

Figure 2a Velocity profiles for various values of M.

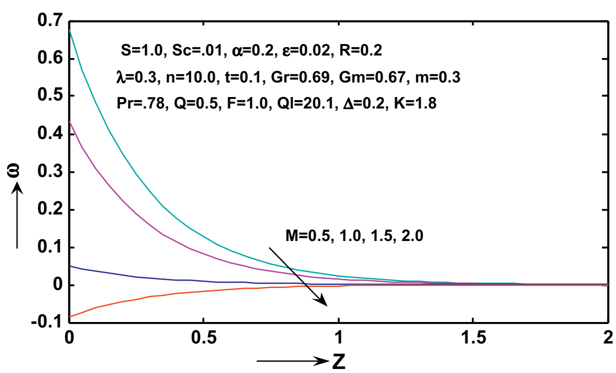

Figure 2b Micro-rotation profiles for various values of M.

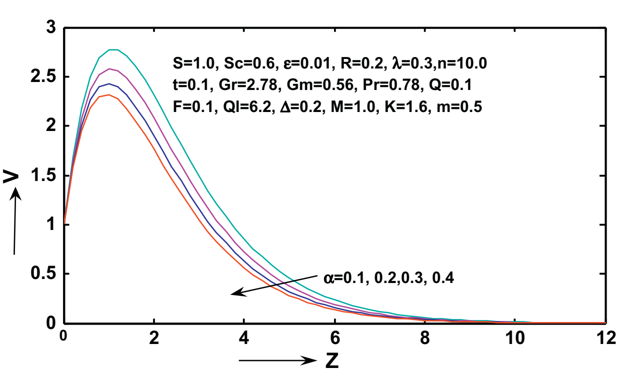

Figure 3a Velocity profiles for various values of α.

that the application of a magnetic field to an electrically conducting fluid produces a dragline force which causes reduction in the fluid velocity. In the case of [33], the magnetic parameter M shows the reverse effect on the velocity profile. This remarkable feature of the velocity profiles in our investigation in due to the presence of transverse magnetic field, Hall current and radiation absorption in the flow field. This phenomenon has a good agreement with the physical realities. Fig. 2b shows that the micro-rotation field decreases with the increase in a magnetic field parameter M within the domain. Further, we see from this figure that the micro-rotation field decreases gradually for the increase of a magnetic field parameter M near the surface.

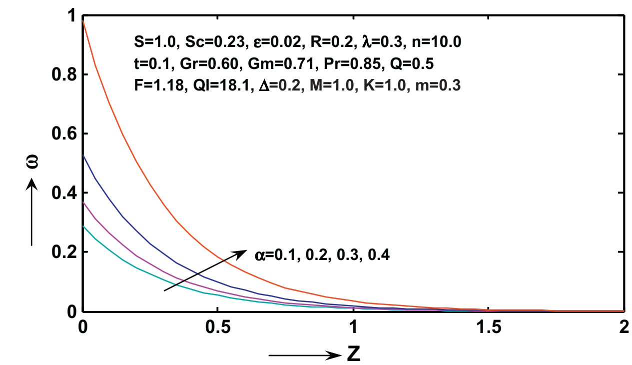

Figure 3b Micro-rotation profiles for various values of α.

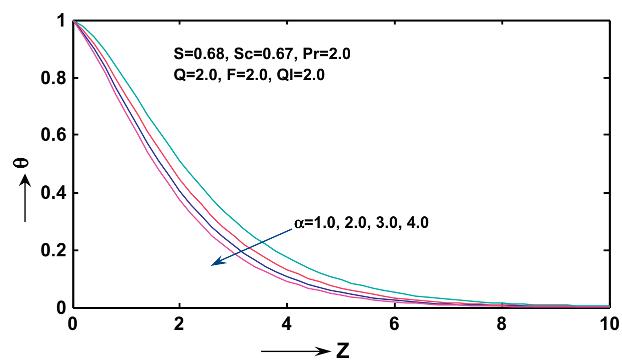

Figure 3c Temperature profiles for various values of α.

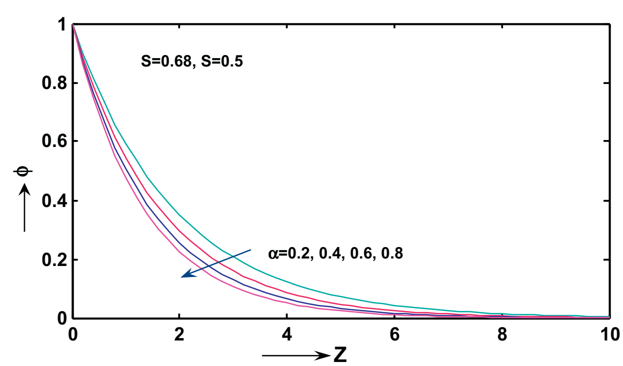

Figure 3d Concentration profiles for various values of α.

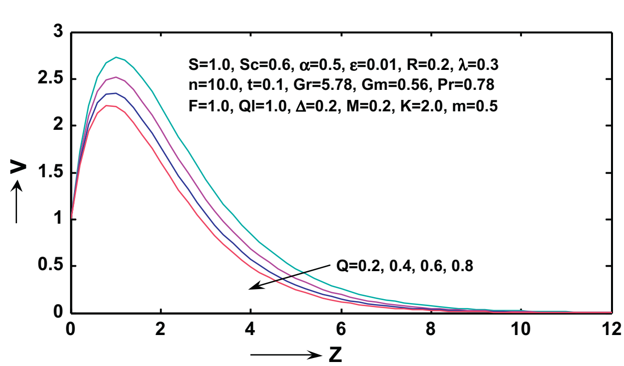

Figure 4a Velocity profiles for various values of Q.

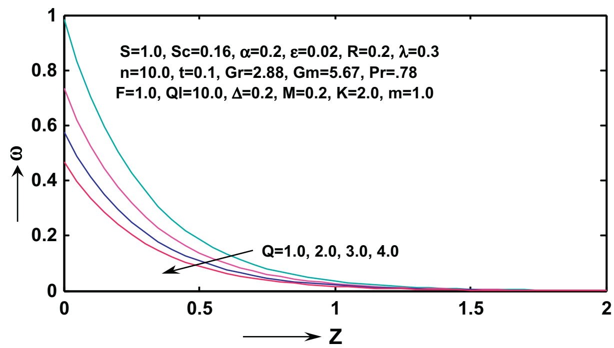

Figure 4b Micro-rotation profiles for various values of Q.

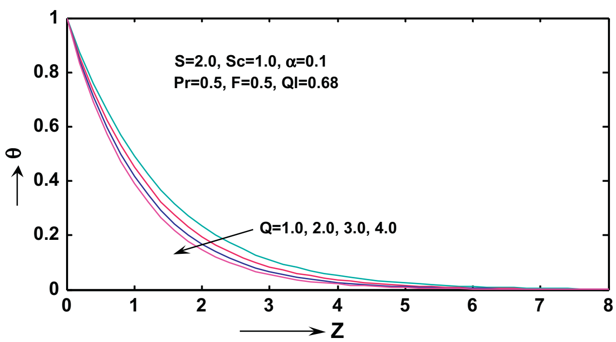

Figure 4c Temperature profiles for various values of Q.

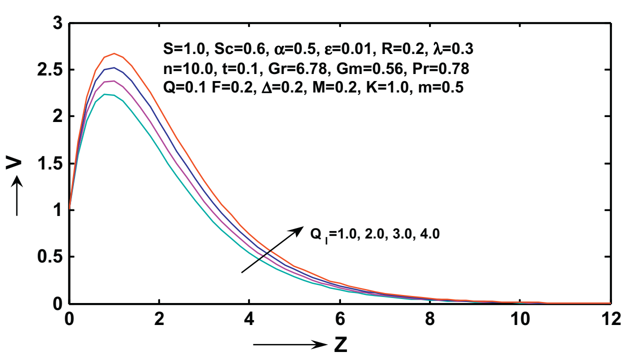

Figure 5a Velocity profiles for various values of Q_l.

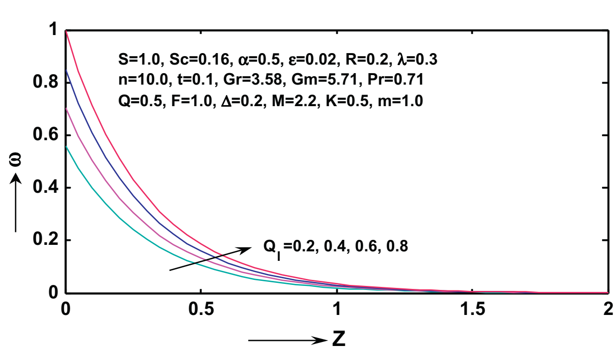

Figure 5b Micro-rotation profiles for various values of Q_l.

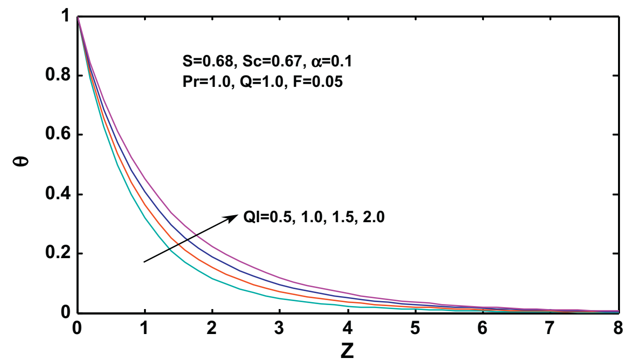

Figure 5c Temperature profiles for various values of Q_l.

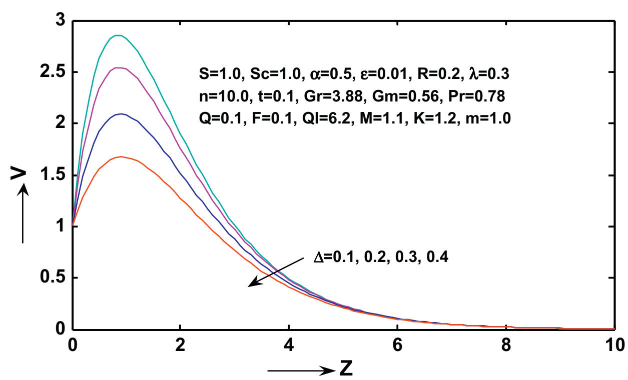

Figure 6a Velocity profiles for various values of Δ.

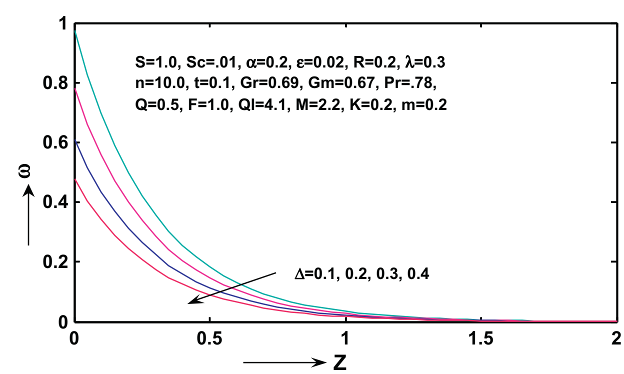

Figure 6b Micro-rotation profiles for various values of Δ.

Figs. 3a, 3b, 3c, and 3d, respectively, show the translation velocity, micro-rotation, temperature, and concentration profiles for different values of the chemical reaction parameter α. It is clear from the figures that the translation velocity, temperature, and concentration profiles decrease with the increase of the parameter α where as the reverse trend is observed in the case of micro-rotation profile. The effect of α on velocity and concentration is reverse in case of [32]. This is due to the dominate role of the Hall current, and radiation absorption in the flow field. These results clearly supported from the physical point of view.

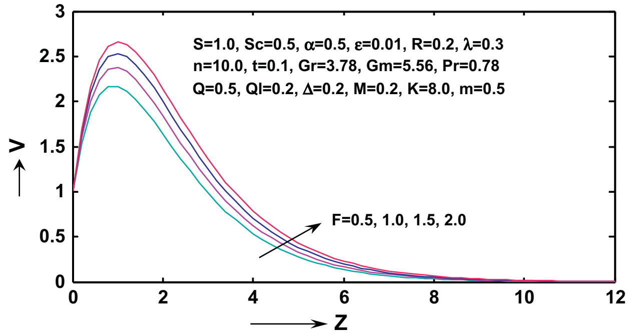

Figure 7a Velocity profile for various values of F.

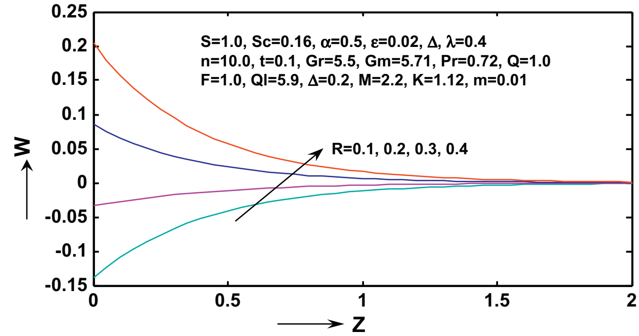

Figure 8b Micro-rotation profiles for various values of R.

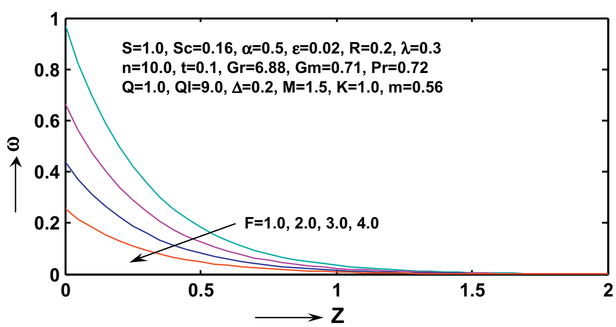

Figure 7b Micro-rotation profile for various values of F.

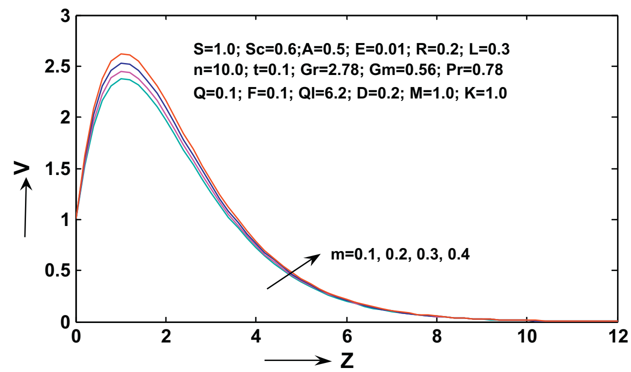

Figure 9a Velocity profiles for various values of m.

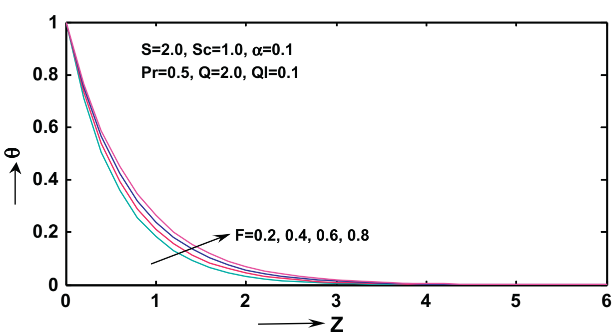

Figure 7c Temperature profile for various values of F.

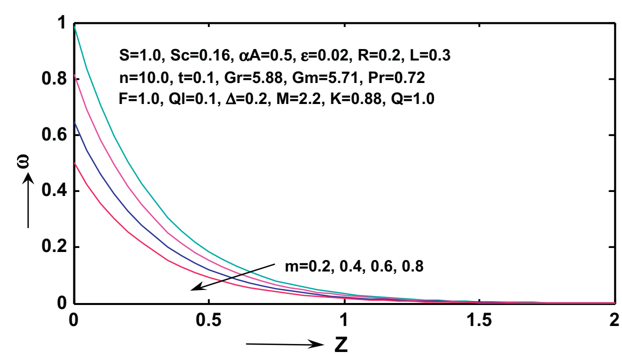

Figure 9b Micro-rotation profiles for various values of m.

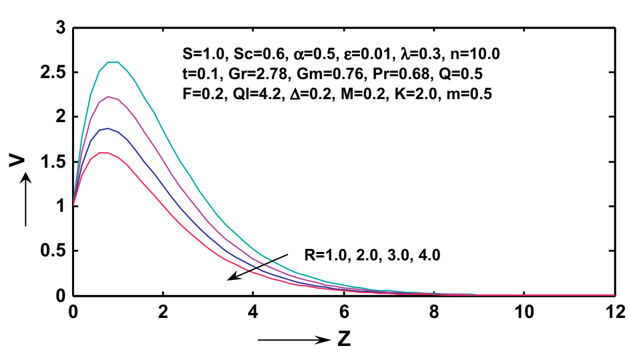

Figure 8a Velocity profiles for various values of R.

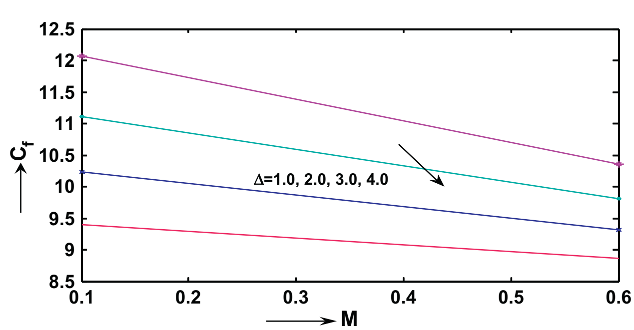

Figure 10a Skin friction for various values of Δ.

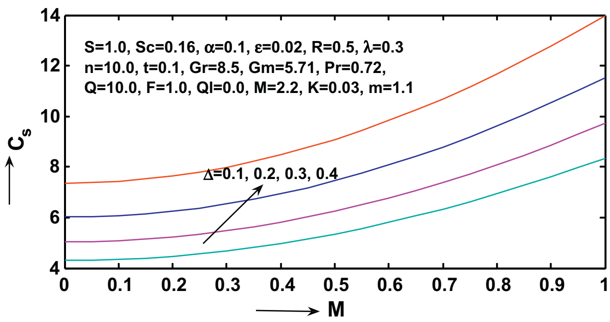

Figure 10b Couple Stress for various values of Δ.

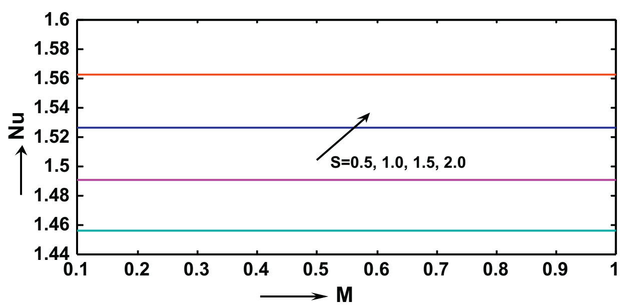

Figure 10c Nusselt number for various values of S.

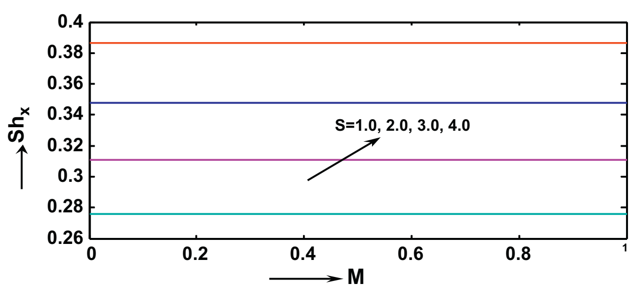

Figure 10d Sherwood number for various values of S.

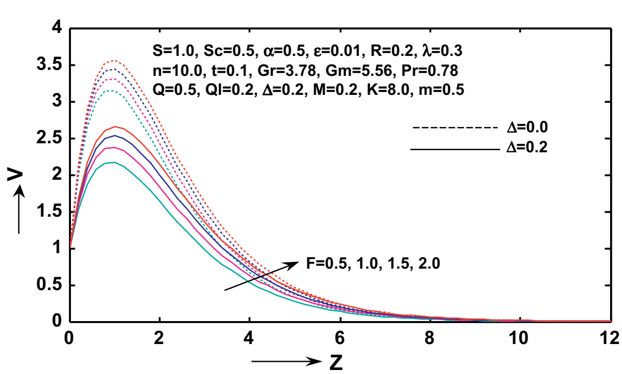

Figure 11a Velocity profiles for various values of F.

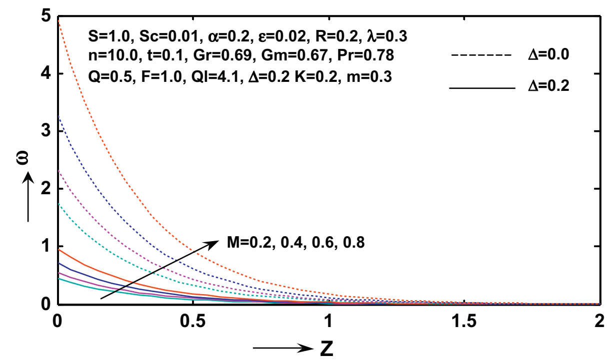

Figure 11b Micro-rotation profiles for various values of M.

Table 1 Comparison of skin-friction, Nusselt number and Sherwood number of the present case with those of Das [33] for different Δ at $S = 1.0$, $Sc = 0.13$, $\alpha = 0.5$, $\varepsilon = 0.02$, $R = 0.2$, $\lambda = 0.02$, $n = 10.0$, $t = 0.1$, $Gr = 12.0$, $Gm = 7.0$, $Pr = 2.81$, $Q = 2.5$, $F = 0.5$, $M = 2.5$, and $K = 10.0$.

Δ	Das [33]			Present results		
	Ql = 0.0, m = 0.0					
	C_f	Nu	Sh	C_f	Nu	Sh
1.0	12.07	1.456	0.276	12.07	1.456	0.276
2.0	11.11	1.456	0.276	11.12	1.456	0.276
3.0	10.23	1.456	0.276	10.23	1.456	0.276
4.0	9.395	1.456	0.276	9.395	1.456	0.276

Figs. 4a, 4b, and 4c, respectively, show the translation velocity, micro-rotation, and temperature profiles for different values of heat generation parameter Q. It is obvious from the graphs, all the translation velocity, micro-rotation and temperature distributions decrease with increase of Q. Heat sink (source) physically implies absorption (evolution) of heat from the surface, which decreases (increases) the temperature in the flow field. Therefore, as heat sink parameter increased, the temperature decreases steeply and exponentially from the plate but heat source parameter enhances it. These findings are similar to those obtained by [32,33].

For various values of Q_l, the profiles of translation velocity, the micro-rotation, and temperature profiles across the boundary layer are shown in Figs. 5a, 5b, and 5c. It is clear from the graphs that all the translation velocity, micro-rotation and temperature increase with an increase of Q_l which is due to the fact that when heat is absorbed the buoyancy force which accelerated the flow.

Figs. 6a and 6b respectively show the effect of Δ on translational velocity and micro-rotation profiles for a stationary porous plate. It shows that both the translational velocity field and micro-rotation decrease as the increasing values of viscosity ratio Δ. These findings are similar to those obtained from [33] but are reverse in case of Ref. [32]. This is due to the presence of radiation parameter in the flow field.

Figs. 7a, 7b, 7c, respectively, show the translational velocity, micro-rotation and temperature profiles for different values of the thermal radiation parameter F. It is observed from figures that both the translational velocity and temperature profiles increase with the increase in radiation parameter (F), but the effect is reversed for micro-rotation distribution. Physically speaking, the thermal boundary layer thickness increases with an increase in the thermal radiation. Thus, it is pointed out that, the radiation should be minimized to have the cooling process at a faster rate.

Figs. 8a and 8b respectively, show the translational velocity and micro-rotation profiles for different values of a rotational parameter R. The results display that, with an increasing of R, the translation velocity decreases, but the magnitude of micro-rotation increases as, R increases as noted in Ref. [32].

Figs. 9a and 9b respectively show the effect of Hall current parameter m on the translational velocity and micro-rotation distributions. We observe from Fig. 9a that the translational velocity field steeply rises up to maximum peaks as the Hall parameter m increases. In Fig. 9b, we see that the micro-rotation profile approaches their classical values when the Hall parameter m becomes large. This means that the micro-rotation profile decreases with increasing values of Hall parameter m.

Figs. 10a, 10b, 10c, 10d display the variation of the local skin friction coefficient C_f, couple stress C_s, Nusselt number Nu and Sherwood number Sh_x with magnetic field M for various values of Δ and S. Figs. 10a and 10b described the behavior of skin friction the coefficient and couple stress with changes in the values of M and Δ. It is observed from Fig. 10a that the local skin friction coefficient is reduced due to increase in the magnetic field strength, as expected since the applied magnetic field tends to impede the flow motion and thus reduces the surface friction force, where as reverse effect is observed in the case of couple stress (from Fig. 10b). On the other hand as Δ increases skin friction decrease and couple stress increase. Figs. 10c and 10d display the heat transfer coefficient and Sherwood number in terms of a magnetic field parameter M corresponding to the different values of S. These figures clearly show that, Nusselt number, and Sherwood number increase with increase in the value of S, whereas M has no effect on Nu and Sh_x. Also from Eqs. (6) and (7) it is clear that the equations are independent of M. So M has no effect on Nu and Sh_x.

Figs. 11a and 11b present the effects of radiation parameter F and magnetic field parameter M on translational velocity and micro-rotation profiles for both Newtonian and polar fluid conditions, respectively. Increasing the radiation parameter and magnetic field parameter has the tendency to increase for both the translational velocity and the micro-rotation profiles. In addition, it is seen that the fluid velocity and micro-rotations are lower for polar fluids than for Newtonian fluids.

4. Conclusions

We analyze the effects of Hall current, rotation and radiation absorption on MHD free convection and mass transfer flow of micropolar fluid past a vertical porous plate. The governing equations were solved analytically by using the perturbation technique. The effects of the various parameters on velocity, micro-rotation, temperature and concentration profiles, etc., are examined. From the present calculations, we may arrive at the following conclusions.

1. The magnetic parameter M retards the velocity of the flow field at all points, due to the magnetic pull of the Lorentz force acting on the flow field. In case of Ref. [33], the above parameter shows wavy character above the axis of a rotation in the plane of the plate. This remarkable feature of the velocity profiles in our investigation is due to the presence of Hall current and radiation absorption. (i.e. the strong magnetic field stabilizes this flow making the wavy character disappears.)
2. The fluid motion is retarded due to chemical reaction. Hence, the consumption of chemical species causes a fall in the concentration field which in turn diminishes the buoyancy effects due to concentration gradients. Hence the flow field is retarded.
3. Due to chemical reaction, the concentration of the fluid decreases. This is because the consumption of chemical species leads to a fall in the species concentration field. But it is reverse in case of Ref. [32]. This is due to the dominate role of the Hall current and radiation absorption in the flow field.
4. The magnitude of skin friction at the plate is found to decrease due to increasing M.
5. The rate of mass transfer Sh_x and rate of heat transfer Nu increase due to increase in S, whereas M exhibits no effect on these profiles.

Acknowledgment

The authors are grateful to the reviewers for their suggestions that extensively enhanced our paper.

Appendix A.

$$a_1 = \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right) + i\left(R + \frac{mM^2}{1+m^2}\right)$$

$$a_2 = \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right) + i\left(R + n + \frac{mM^2}{1+m^2}\right),$$

$$a_3 = \left(\frac{M^2}{1+m^2} + \frac{1}{K}\right) + i\left(R - n + \frac{mM^2}{1+m^2}\right)$$

$$m_1 = \frac{SSc + \sqrt{(SSc)^2 + 4\alpha Sc}}{2}$$

$$m_2 = \frac{3SPr + \sqrt{(3SPr)^2 + 12Q(3+4F)}}{2(3+4F)}$$

$$m_3 = \frac{S + \sqrt{S^2 + 4a_1(1+\Delta)}}{2(1+\Delta)}$$

$$m_4 = \frac{S + \sqrt{S^2 + 4in\lambda}}{2\lambda} \qquad m_5 = \frac{S + \sqrt{S^2 + 4a_2(1+\Delta)}}{2(1+\Delta)}$$

$$m_6 = \frac{S + \sqrt{S^2 - 4in\lambda}}{2\lambda} \qquad m_7 = \frac{S + \sqrt{S^2 + 4a_3(1+\Delta)}}{2(1+\Delta)}$$

$$A_1 = \frac{-3Q_l Pr}{(3+4F)m_1^2 - 3SPrm_1 - 3Q}$$

$$A_3 = \frac{-(Gm + GrA_1)}{(1+\Delta)m_1^2 - Sm_1 - a_1}$$

$$A_4 = \frac{-GrA_2}{(1+\Delta)m_2^2 - Sm_2 - a_1}$$

$$A_6 = \frac{\Delta S\lambda\{A_3(m_1 - m_3) + A_4(m_2 - m_3) + m_3\}}{(2+\Delta)S^2 - 2\lambda(S^2 + \lambda a_1) + \Delta S\lambda m_3}$$

$$A_7 = \frac{\Delta m_4 m_5}{(2+\Delta)m_4^2 - 2(Sm_4 + a_2) + \Delta m_4 m_5}$$

$$A_9 = \frac{\Delta m_6 m_7}{(2+\Delta)m_6^2 - 2(Sm_6 + a_3) + \Delta m_6 m_7}$$

$$A_2 = 1 - A_1, \quad A_5 = 1 - A_3 - A_4 - A_6$$

$$A_8 = 1 - A_7 \text{ and } A_{10} = 1 - A_9$$

$$B_1 = \frac{i\{A_3(m_3 - m_1) + A_4(m_3 - m_2) - m_3\}\{(1+\Delta)S^2 - \lambda S^2 - a_1\lambda^2\}}{(2+\Delta)S^2 - 2\lambda(S^2 + \lambda a_1) + \Delta S\lambda m_3}$$

$$B_2 = \frac{-im_5\{(1+\Delta)m_4^2 - Sm_4 - a_2\}}{(2+\Delta)m_4^2 - 2(Sm_4 + a_2) + \Delta m_4 m_5}$$

$$B_3 = \frac{-im_7\{(1+\Delta)m_6^2 - Sm_6 - a_3\}}{(2+\Delta)m_6^2 - 2(Sm_6 + a_3) + \Delta m_6 m_7}$$

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